Cell Current Noise Analysis – a Worked Example

Regarding

Examination of Statistical and Spectral Characteristics

of

Noise present in Fuel Cell Current Waveforms

Caused by

Bubble Formation and Release

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Purpose of Cell Current Example Noise Analysis:

1. Demonstrate how to analyze the statistics and spectral energy distribution from “noise” current waveforms from fuel cell devices.

2. Demonstrate how to use time series analysis results to aid understanding of physical processes in fuel cell electrode structures

Summary of Results for Cell Noise Current Analysis:

1. Noise patterns have well defined morphology that relate to bubble accumulation and release.

2. Noise patterns are loosely determined by average cell current and operation time.

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Nature of the Measurement:

1. Current is measured by a potentiostat

2. The measured data is entered into an Excel spread sheet

Nature of the Analysis :

1. The “raw” time series is detrended to produce a zero mean time series with the very long term variations removed.

2. The detrended time series is divided up into three “chunks” so that variation of extracted noise parameters during the “beginning”, “middle” and “end” of the test run can be compared. These chunks consist of 512 (512 = 2^9) data points (a power of 2 - is required by fast Fourier analysis algorithms) and represent 8.5 minutes of data collection.

3. Descriptive statistics (probability distribution function histogram – moments - Power Spectral Density (PSD) and Autocorrelation Function are then calculated and plotted on “Summary Report” sheets in an Excel workbook.

4. Examination of the Summary Reports allows information about gas formation in the electrode structures and fluid manifolds to be inferred.

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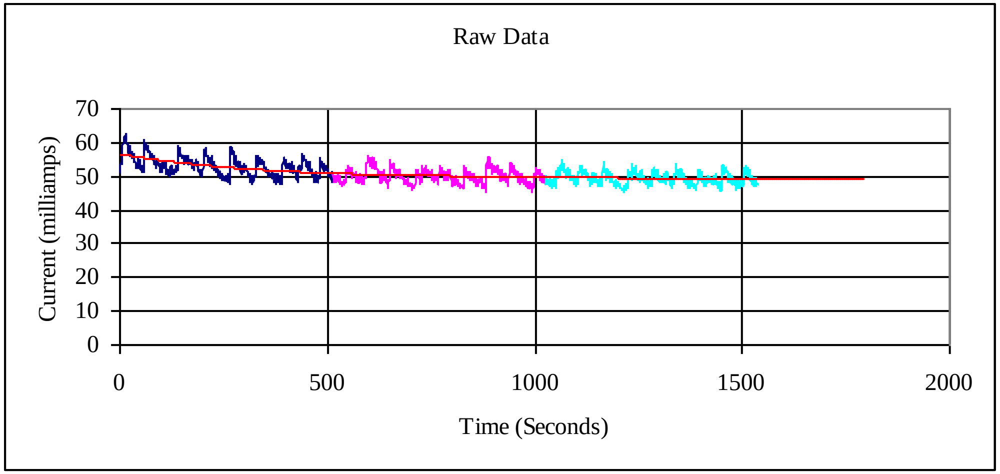

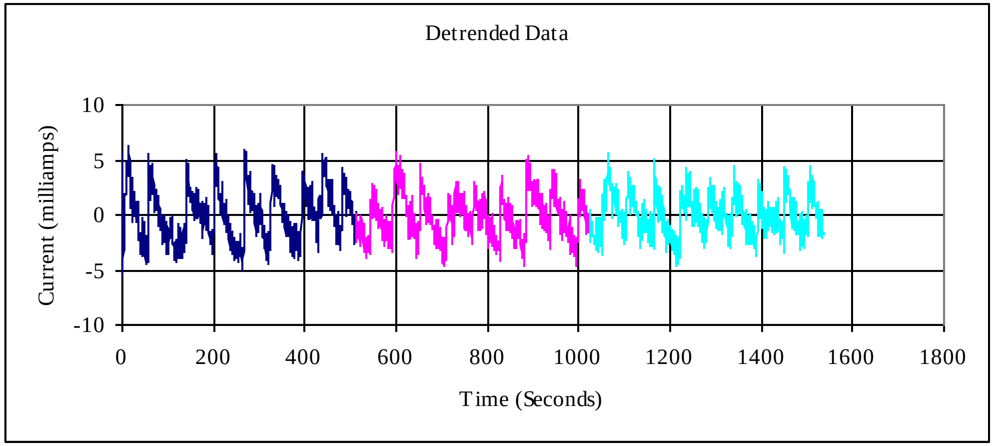

Typical Cell Current Waveform Before and After "Detrending"
by Fitting a Fourth Order Polynomial to the "Raw" Data Time Series

(The time series is split into three "chunks" to allow variations in parameter values during the course of an experiment)

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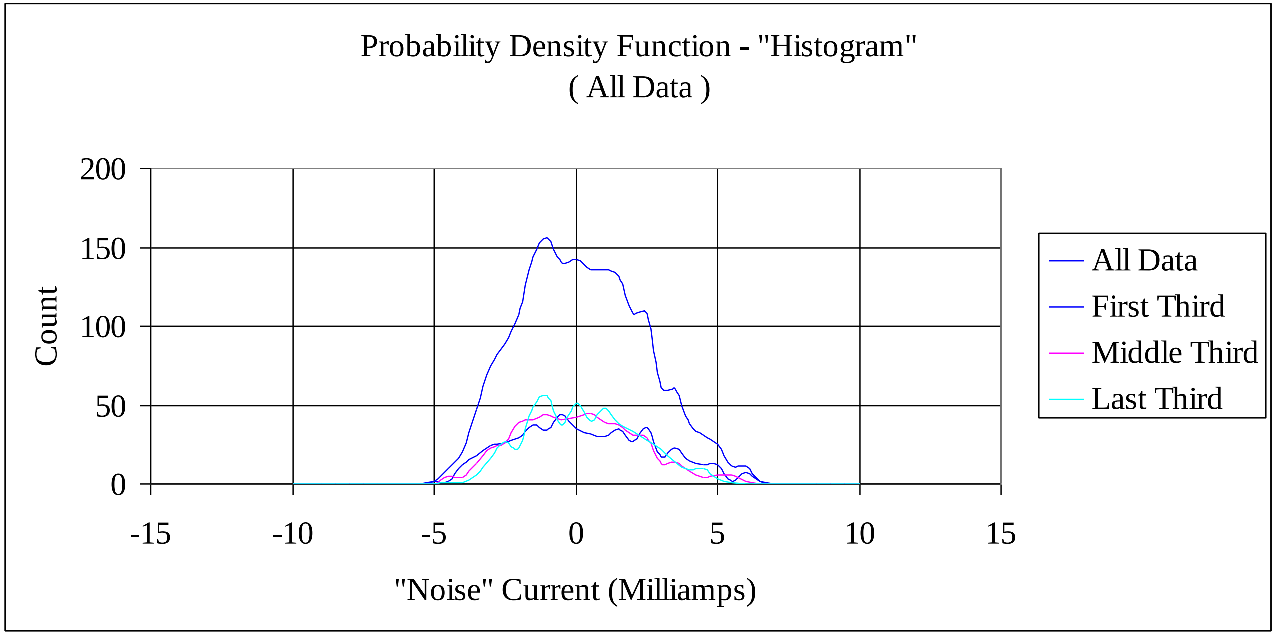


Histogram Approximation to the Probability Distribution Function

Notice that the PDF is clearly not “Normal – Gaussian” – it is more full in the middle and does not have long “tails”. Also notice that there is a fair amount of “skew”.

The First Through the Fourth Moments of a Probability Distribution Function

First Moment = the Mean = $\int_{-\infty}^{\infty} x \cdot F(x)\,dx$ "Center of Gravity"

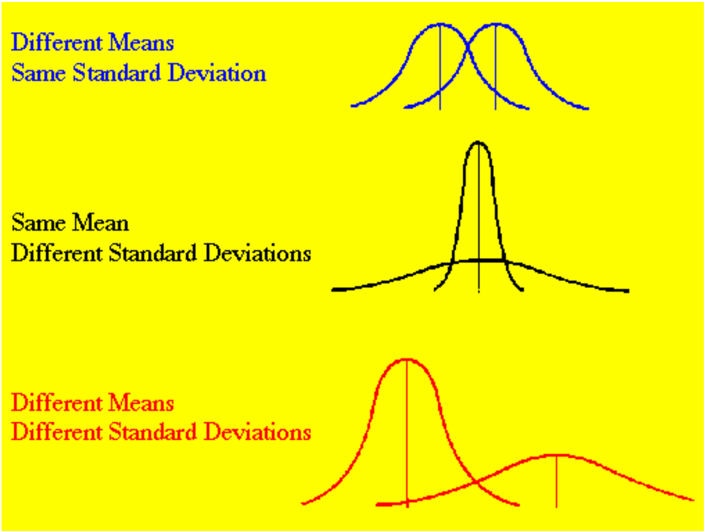
Second Moment = the Variance = $\int_{-\infty}^{\infty} x^2 \cdot F(x)\,dx$ "Radius of Gyration"

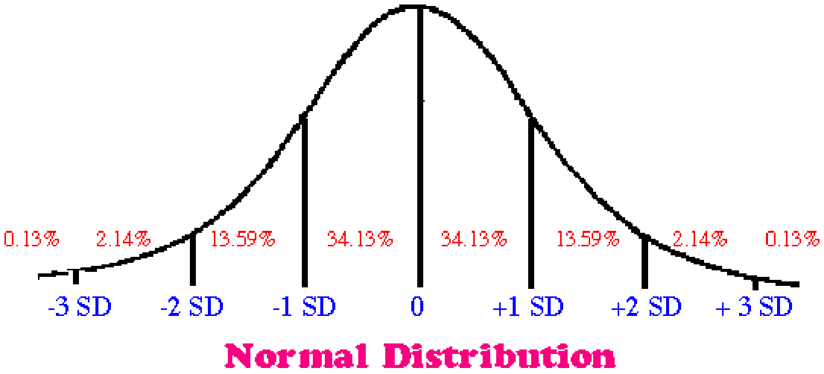
Third Moment = the Skew = $\int_{-\infty}^{\infty} x^3 \cdot F(x)\,dx$ "Measure of Asymmetry"

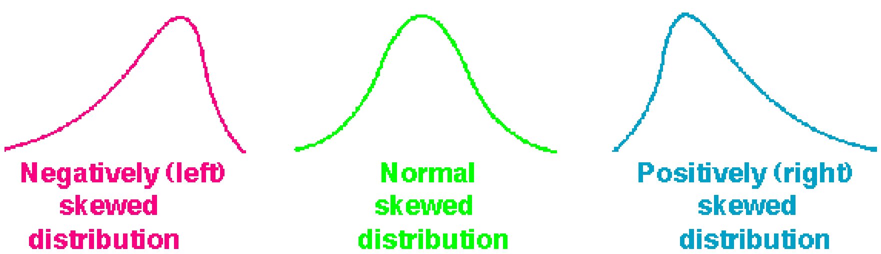
Fourth Moment = the Kurtosis = $\int_{-\infty}^{\infty} x^4 \cdot F(x)\,dx$ "Measure of Central Tendency"

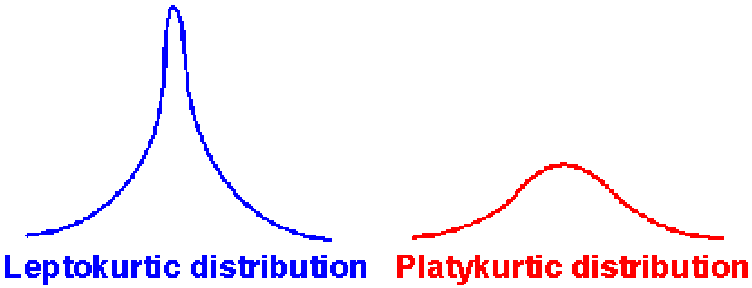
These four parameters quantitatively describe the shape, spread and location of a probability distribution function. Each parameter is the integrated result of all the data in a particular time series and thus may be used to compare the histograms from similar but different fuel cell noise current waveforms. Use of these parameters represents the classical statistical analysis approach to knowledge inference from time series data consisting of information submerged in random data.

The First Through the Fourth Moments of a Probability Distribution Function









Kurtosis - "Measure of skinniness"

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	All Data	First Third	Middle Third	Last Third
Mean	9.3E-06	0.109	-0.155	0.0265
Standard Error	0.05	0.109	0.094	0.0851
Median	-0.10	-0.063	-0.215	-0.0852
Standard Deviation	2.20	2.465	2.125	1.9246
Sample Variance	4.85	6.076	4.515	3.7041
Kurtosis	-0.45	-0.640	-0.326	-0.5075
Skewness	0.25	0.210	0.302	0.2446
Range	11.77	11.769	10.662	10.1077
Minimum	-5.32	-5.323	-4.794	-4.7200
Maximum	6.45	6.447	5.869	5.3877
Sum	0.02	56.076	-79.361	13.5587
Count	1792.00	513.000	512.000	512.0000
Largest(1)	6.45	6.447	5.869	5.3877
Smallest(1)	-5.32	-5.323	-4.794	-4.7200
Confidence Level(95.0%)	0.10	0.214	0.184	0.1671

Descriptive Statistics

As expected by examination of the histogram, the Kurtosis and Skewness are non-zero and significant (the kurtosis < 0 means "flat topped" Kurtosis > 0 means "pointy topped". The kurtosis = 0 for the normal-Gaussian distribution)

Note: The "Mode" is not given because, due to very high data acquisition resolution and subsequent numerical manipulation, no two data points had the exact same value

The Power Spectral Density Function

Fourier Transform = Magnitude and Phase Spectrum = $F(s) = \int_{-\infty}^{\infty} f(x) \cdot e^{-i \cdot 2 \cdot \pi \cdot s} \, dx$

Inverse Fourier Transform = Real or Complex Time Series = $f(x) = \int_{-\infty}^{\infty} F(s) \cdot e^{-i \cdot 2 \cdot \pi \cdot \omega} \, ds$

Power Spectral Density = PSD = $\int_{-\infty}^{\infty} \left(\left|F(s)\right|\right)^2 ds$ or $\int_{-\infty}^{\infty} F(s) \cdot F(s)^{\phi} \, ds$

where $F(s)^{\phi}$ is the complex conjugate of F(s) and s is the complex frequency (j* ω)

f(x) is the time series to be analyzed and F(s) is the complex (mag and phase) Fourier Transform of the time series

The Power Spectral Density Function tells us at which frequencies there is energy within the time series that we are analyzing. A plot of amplitude, power or energy vs. frequency is called a “Spectrogram”

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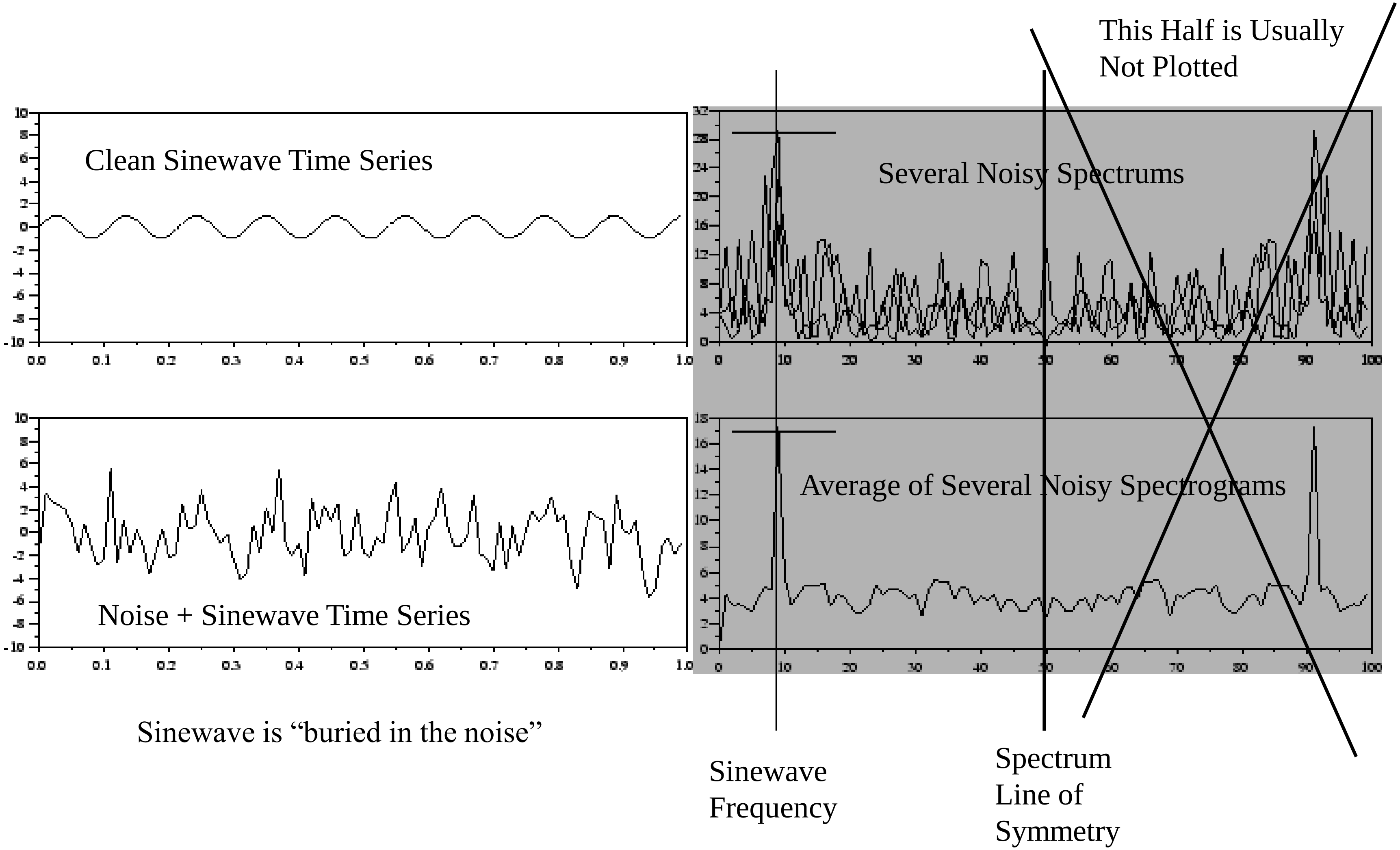
The Power Spectral Density Function for a Noised Sine Wave



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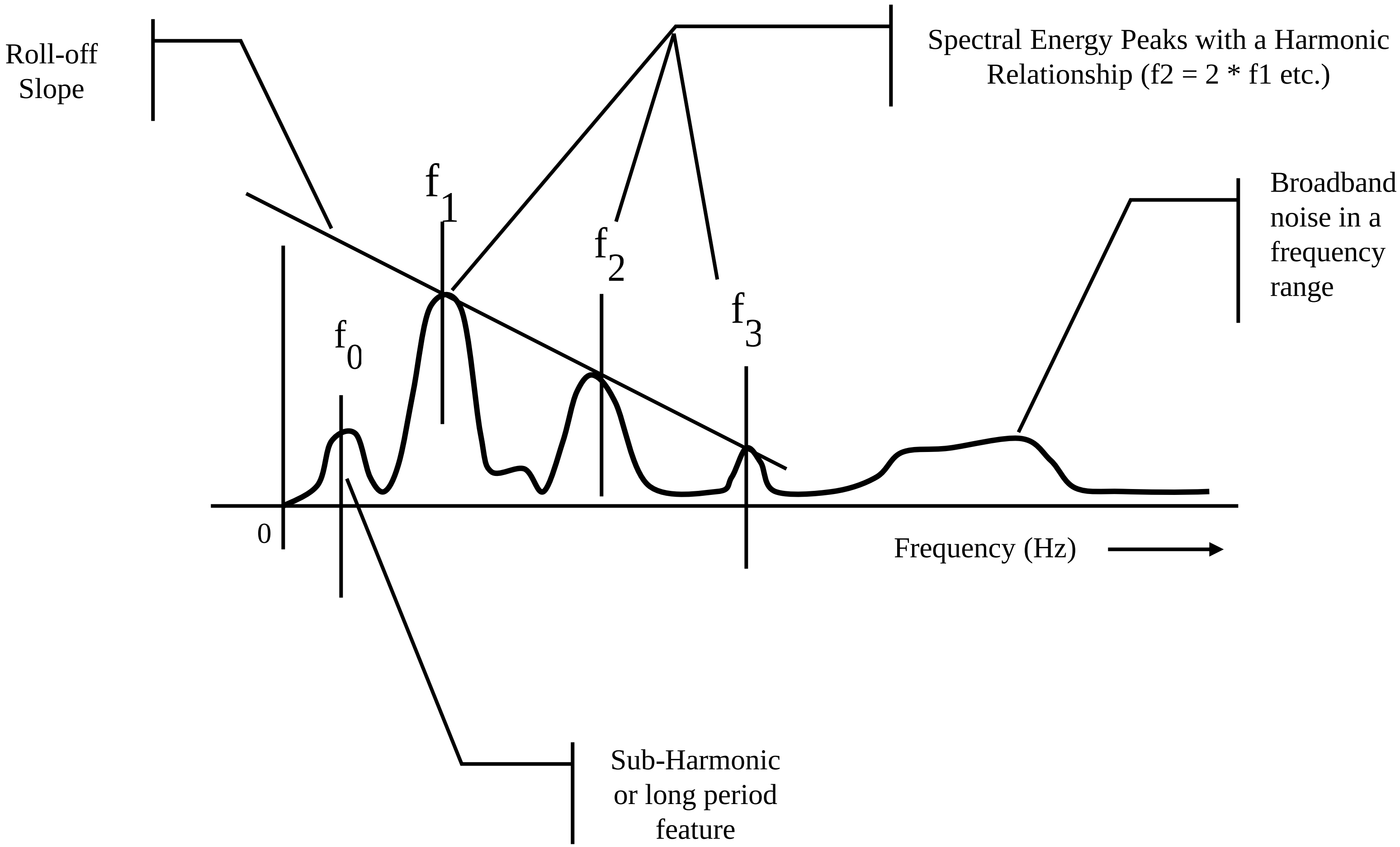
What to Look for When Using the Power Spectral Density Function



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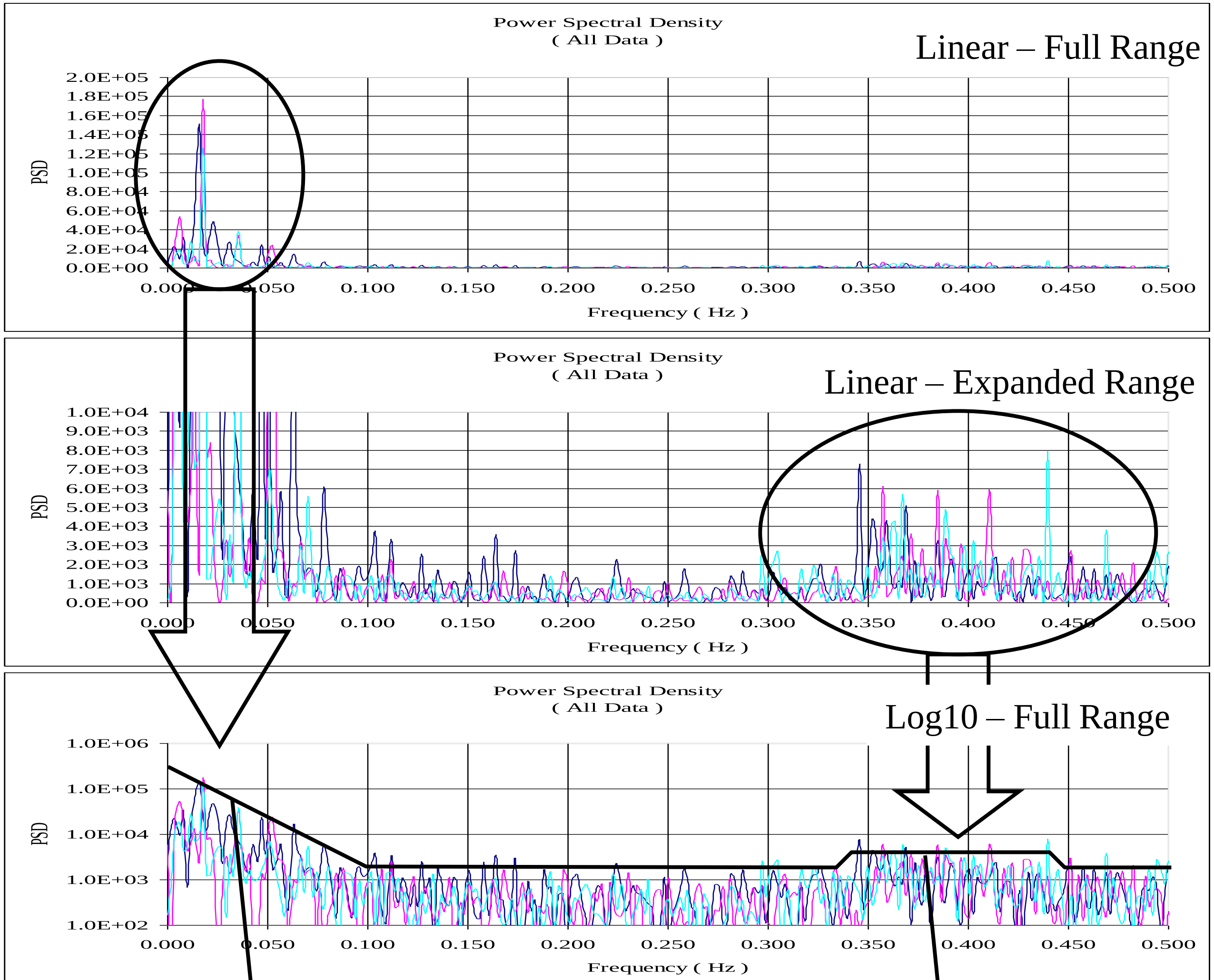
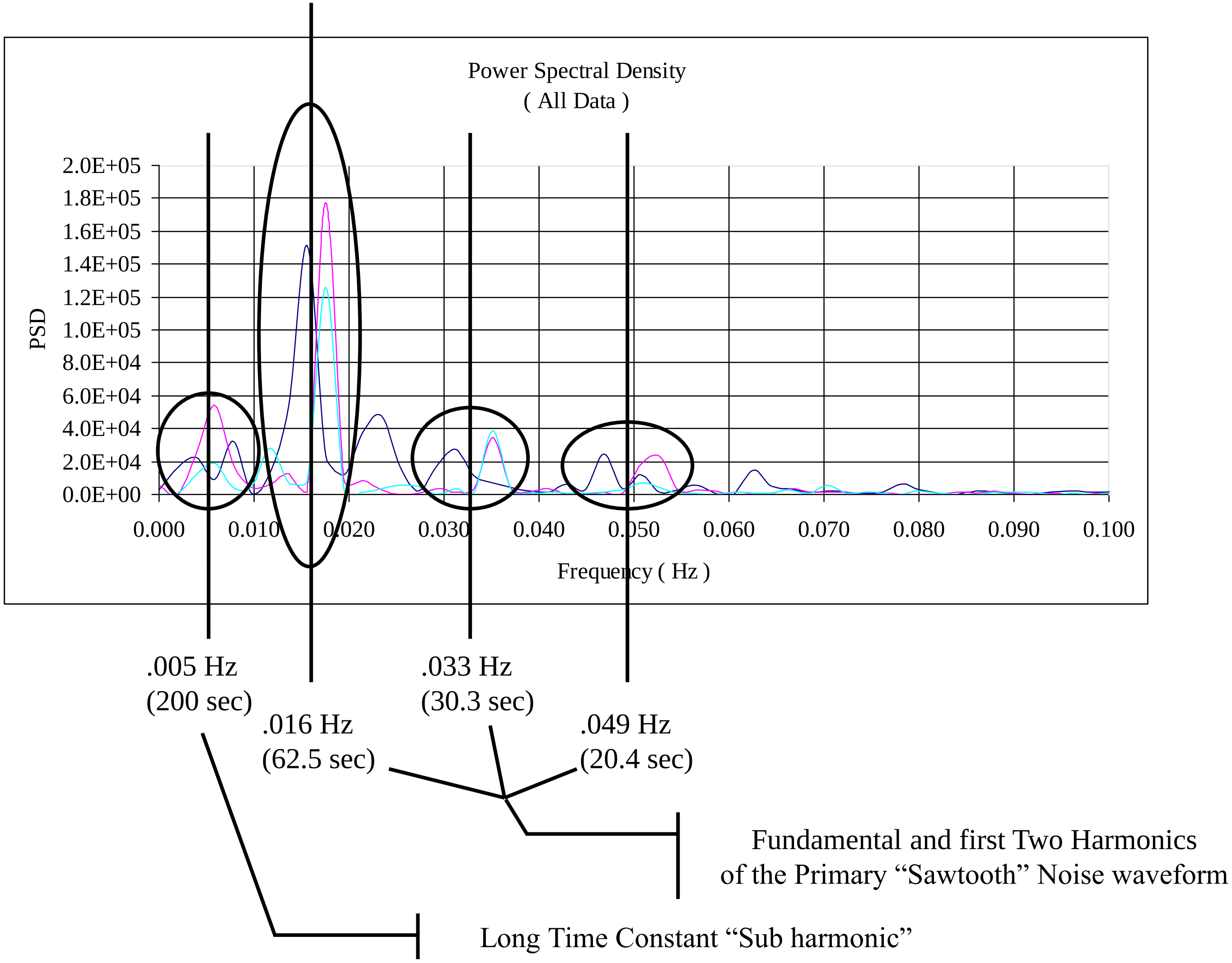


Power Spectral Density

"Noise Source with Roughly 20 dB per Decade Power Density "Roll-off"

High Frequency Wideband "Noise" Source



Power Spectral Density – Magnified Plot of Low Frequency Range

The Autocorrelation Function

$$\text{Auto-Correlation Function} \quad = \quad ACF(\tau) \quad = \int_{-\infty}^{\infty} F(\tau + x)\cdot F(\tau)^{\phi}\, dx \qquad \text{or} \qquad \int_{-\infty}^{\infty} PSD\cdot e^{-i\cdot 2\cdot \pi\cdot \omega}\, ds$$

where $F(\tau)^{\phi}$ is the complex conjugate of $F(\tau)$, τ is the relative correlation time delay and s is the complex frequency ($j^{*}\ \omega$)

The Autocorrelation Function measures how similar a time series is to itself when compared at different relative time delays. Because the Autocorrelation Function is the inverse Fourier transform of the Power Spectral Density Function, it represents the same information … but … in a different way.

The PSD relates the time series and its energy at different frequencies. The ACF relates the time series to a time delayed copy of itself. Because each is the Fourier transform of the other, a feature in the time series that repeats itself at a fairly regular time intervals will be represented by a peak in the Autocorrelation function at a time delay equal to the repetition interval. The same feature will appear in the Power Spectral Density plot as a "peak" at a frequency equal to the inverse of the time delay (freq = 1 / time).

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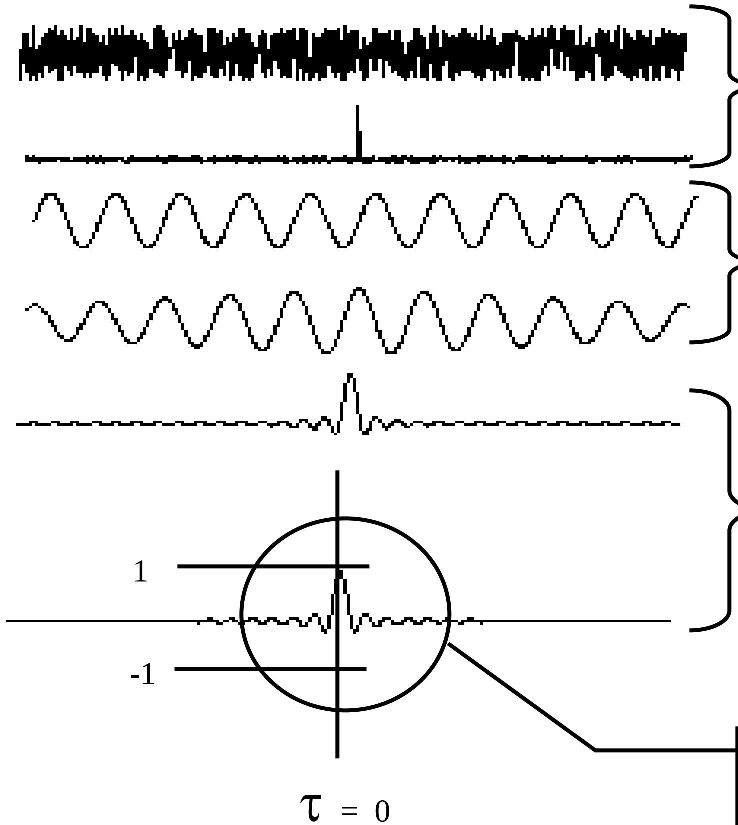
The Autocorrelation Function



- Random noise is similar to itself, and in phase, only with no time shift at all
- so its correlation function is a spike
- periodic signals go in and out of phase as they are time shifted
- so their correlation functions are periodic
- signals that last only a short while are only similar while they last
- so their correlation functions are short

Magnified and explained on the next page

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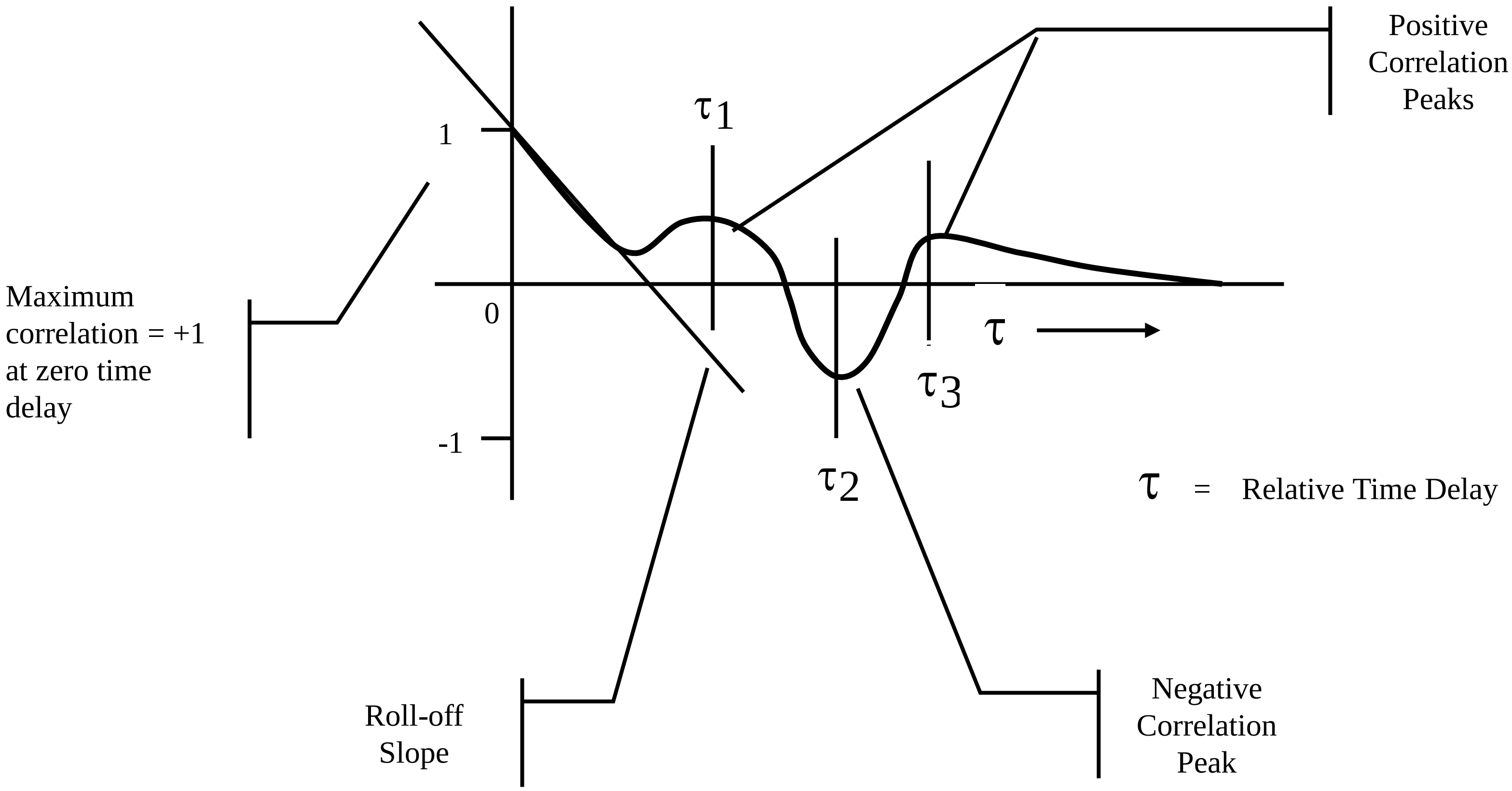
What to Look for When Using the Autocorrelation Function



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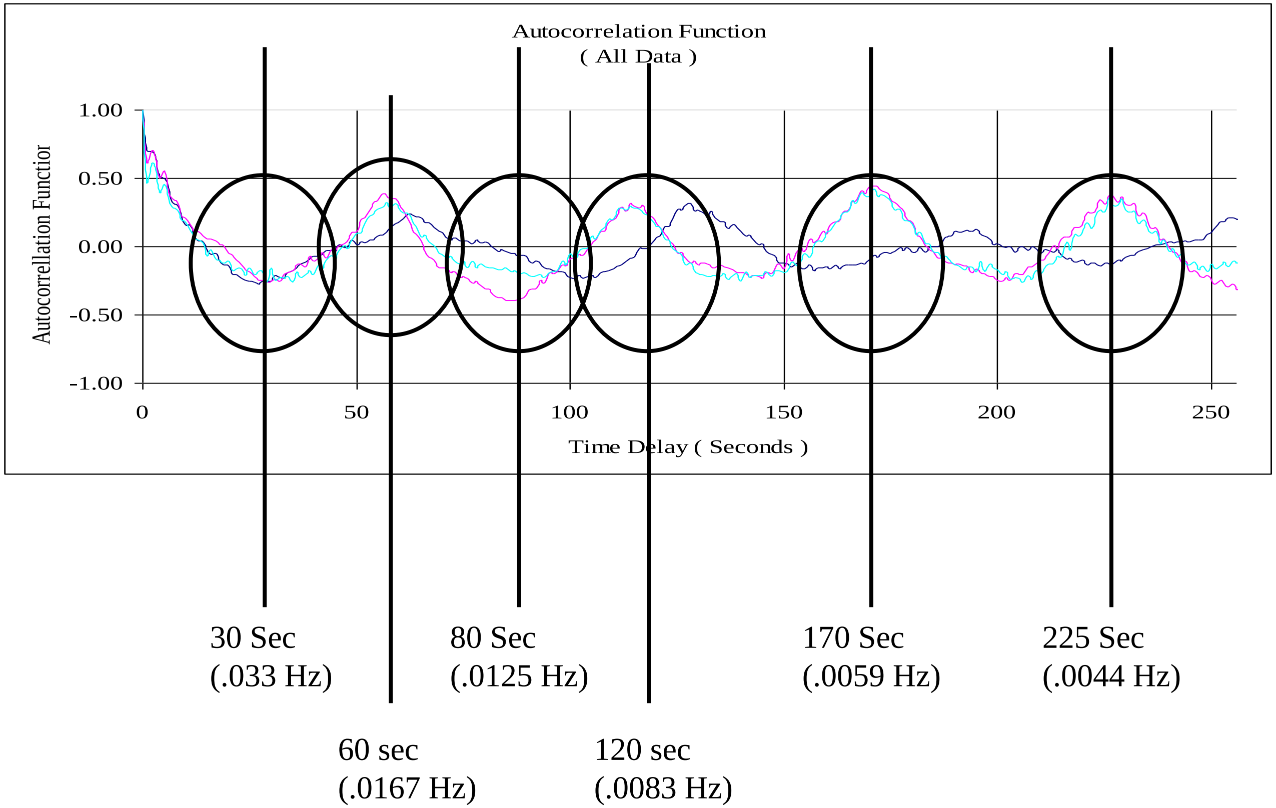
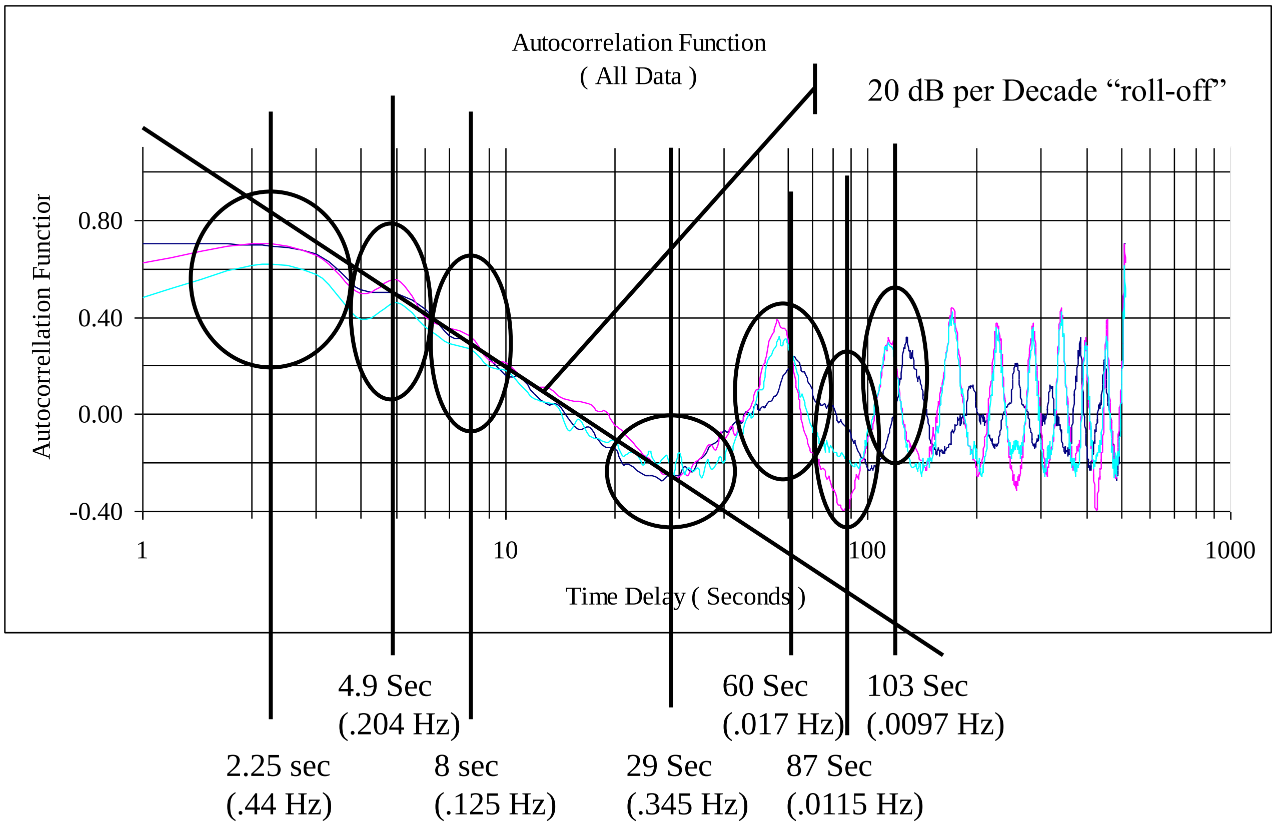


Autocorrelation Function

The positive and negative peaks (autocorrelation maxima for "in phase" and "out of phase" waveform features) are at time delays that are nominal inverses of the frequency peaks in the Power Spectral Density Function. At zero time delay, the autocorrelation function is always 1.0.



Autocorrelation Function – Log10 Time Delay

The positive and negative peaks (autocorrelation maxima for "in phase" and "out of phase" waveform features) are at time delays that are nominal inverses of the frequency peaks in the Power Spectral Density Function. At zero time delay, the autocorrelation function is always 1.0.

Summary:

1. These powerful statistical measures of cell current "noise" analysis may be helpful in determining various bubble production, release and extraction mechanisms.

2. These tools may be helpful in making comparisons of surfactants.

3. These tools can give a quick check on whether a fuel cell test run has performed in a reasonably correct manner.

Additional Comments:

1. Once the process is automated, it is very easy to run a battery of statistical tests on all fuel cell run data. Since we have a large library of data already in hand, we can quickly learn a lot about how bubble induced cell current noise behaves under a wide range of experimental conditions.

2. The methods presented in this tutorial are all well established and commonly used by statisticians and reliability engineers.

3. The combination of moments, spectra and autocorrelation functions are powerful tools that can help us resolve difficult questions.